BILATERAL ENHANCERS

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ABSTRACT
Ten years ago the concept of bilateral filtering (BF) became popular in the image processing community. The core of the idea is to blend the effect of a spatial filter, as e.g. the Gaussian filter, with the effect of a filter that acts on image values. The two filters act on orthogonal domains of a picture: the 2D lattice of the image support and the intensity (or color) domain. The BF approach is an intuitive way to blend these two filters giving rise to algorithms that perform difficult tasks requiring a relatively simple design. In this paper we extend the concept of BF, proposing the Bilateral Enhancers (BE). We show how to design proper functions to obtain an edge-preserving smoothing and a selective sharpening. Moreover, we show that the proposed algorithm can perform edge-preserving smoothing and selective sharpening simultaneously in a single filtering.

Index Terms— Bilateral Filtering, noise removal, image enhancement, sharpening.

1. INTRODUCTION
The Bilateral Filtering (BF) has been introduced in [1] as a technique for edge-preserving image smoothing and later has been successfully used in diverse image processing and computer graphics applications, among others [2, 3, 4]. There are plenty of papers that provide theoretical explanation of the BF properties and connections with nonlinear diffusion, adaptive smoothing and mean shift [5, 6].

The formulation of BF is:

\[ h(x) = k^{-1}(x) \int_{\Omega(x)} f(\xi) \ c(\xi, x) \ s(f(\xi), f(x)) \ d\xi \]  

(1)

with the normalization factor

\[ k(x) = \int_{\Omega(x)} c(\xi, x) \ s(f(\xi), f(x)) \ d\xi \]  

(2)

where \( f \) is the input image, \( h \) is the output image, \( \Omega(x) \) is a subset of the input image \( f \), \( x \) is the coordinate of a generic pixel in the image, \( \xi \) is the integration variable representing pixels coordinates. The function \( c \) operates on the spatial domain while \( s \) operates on the range domain.

It is important to note that the BF performs a weighted average of image values \( f \) in the neighborhood \( \Omega(x) \) independently of the specific design of \( c \) and \( s \). This fact is evident if we consider that the normalization factor \( k(x) \) is the sum of the weighting coefficients used in formula (1).

However, the design of both \( c \) and \( s \) is an important issue. Usually, an isotropic function is used for \( c \), such that it directly acts on the distance between \( x \) and \( \xi \), i.e. \( c(||x - \xi||) \). Regarding \( s \), an unbiased function is commonly used; it can be thus reformulated as \( s(f(x) - f(\xi)) \). Obviously, the BF is not limited to these two kind of functions. A paradigmatic case of unbiased BF is provided by the use of a Gaussian function on the image domain, \( c(\xi, x) = \exp \left( -\frac{||x - \xi||^2}{2\sigma_c^2} \right) \) and a Gaussian function on the range domain, \( s(f(\xi), f(x)) = \exp \left( -\frac{(f(x) - f(\xi))^2}{2\sigma_s^2} \right) \). This design provides and efficacious BF that performs edge-preserving image smoothing using two parameters (\( \sigma_c \) and \( \sigma_s \)) that are easy to understand and tune [1].

In this paper we argue that the idea of coupling the filtering in the spatial and intensity domain can be successfully applied in order to both remove noise and enhance edges. Thus we propose a method, called Bilateral Enhancers, that at the same time performs edge-preserving smoothing and selective sharpening.

2. BILATERAL ENHANCERS
Following the same notation used in the BF, we now show the modifications applied to formula (1) in order to obtain the BE. First of all, we consider that \( \xi = x \) is a special case in the integration and rewrite formula (1) as:

\[ h(x) = k^{-1}(x) \left( g f(x) + \int_{\Omega(x) \setminus x} f(\xi)c(\xi, x)s(f(\xi), f(x))d\xi \right) \]  

(3)

where \( g = c(x, x)s(f(x), f(x)) \). This step does not change the nature of the BF but highlights that the neighborhood values contributes to the integral depending on the spatial and intensity differences between the center pixel \( x \) and \( \xi \) while, for
the special case $\xi = x$, the contribution of $c$ and $s$ can be summarized into the constant $g$.

The next step is to remove the parts of the BF that force it to perform a weighted average of the neighborhoods. This two parts are the normalization factor $k(x)$ and the intensity $I(f(x))$ in the integral. Removing these two parts we obtain the formulation of the Bilateral Enhancers:

$$j(x) = gf(x) + \int_{\Omega(x) \cap x} c(\xi, x) \ p(f(x), f(\xi)) \ d\xi$$

where, for notational reasons, $s$ has been renamed to $p$.

Within this formulation, $c$ acts in the same way as in the BF, so that it weights the importance of neighborhood pixels depending on their spatial position. On the other hand, despite the formal similarity, $p$ is significantly different from $s$.

The formulation of the BE is apparently similar to the Non-linear Gaussian filters proposed in [7]. Differently from the Non-linear Gaussian filters, in the BE the normalization factor has been removed. As commented above, removing the normalization factor gives the proposed algorithm the ability to perform edge-preserving and contrast enhancement simultaneously in one single pass. Recently, in [8] the authors proposed an Adaptive BF (ABF) capable of performing smoothing and sharpening. In the ABF, the edge slope is enhanced by transforming the histogram via a range filter with adaptive offset $\xi$ and width (by means of an adaptive standard deviation $\sigma_r$). Our approach is completely different since it is not an extension of the BF approach and, moreover, to perform selective sharpening we do not rely on a local image analysis neither pixel classification.

### 2.1. Design of an edge-preserving smoothing

An edge-preserving smoothing has to be able, as in the BF, to discriminate between small intensity variations, to be smoothed, and big variations, usually representing edges, to be preserved. In the BE there is no direct way to perform a weighted average of the neighborhood, but the same effect can be obtained by an appropriate design of the function $p$. We start setting $c(\xi, x) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp\left(-\frac{|x-\xi|^2}{2\sigma_c^2}\right)$ as in the BF, since the meaning of $c$ is identical. Then, it is easy to demonstrate (see Appendix A) that, setting $g = 1$ and $p(\cdot, \cdot) = f(x) - f(\xi)$, the BE reduces to a Gaussian filter$^1$.

Now, to design an edge-preserving smoothing, we need that the Gaussian filtering is performed if the intensity difference is sufficiently small. To this aim, we can set

$$p_e(\cdot, \cdot) = \eta_e (f(x) - f(\xi)) \left(1 - e^{\left(-\frac{(\eta(x)-\eta(\xi))^2}{2\sigma_e^2}\right)}\right)$$

where $\eta_e$ and $\sigma_e$ has similar meaning as for the edge-preserving smoothing. In this case, for small intensity differences, $p_e$ is close to zero, thus no enhancement is performed. This approach is meant not to enhance the noise eventually present in the image. Figure 2 shows an example of the filtering using $\eta_e = 1$, $\sigma_e = 1$ and $\sigma_e = 0.08$.

### 2.2. Design of a selective sharpening

As commented above, the proposed method, by means of a proper $p(\cdot, \cdot)$, can perform sharpening. First of all, keeping $c$ as before, if we set $p(\cdot, \cdot) = f(x) - f(\xi)$, thus with inverted signs with respect to the Gaussian smoothing, it can be demonstrated that the BE approximates to a DoG-based sharpening (see Appendix B). As done before, if we want the sharpening to act mainly on large differences, avoiding to enhance small difference due e.g. noise, we can design

$$p_e(\cdot, \cdot) = \eta_e (f(x) - f(\xi)) \left(1 - e^{\left(-\frac{(\eta(x)-\eta(\xi))^2}{2\sigma_e^2}\right)}\right)$$

where $\eta_e$ and $\sigma_e$ has similar meaning as for the edge-preserving smoothing. In this case, for small intensity differences, $p_e$ is close to zero, thus no enhancement is performed. This approach is meant not to enhance the noise eventually present in the image. Figure 2 shows an example of the filtering using $\eta_e = 1$, $\sigma_e = 1$ and $\sigma_e = 0.08$.

### 2.3. Simultaneous smoothing and enhancement

The proposed method is able to perform edge-preserving smoothing and selective sharpening simultaneously. Moreover, the design of the proper function $p$ is trivial. It is sufficient to set $p$ as the sum of the two $p$ functions designed in formula (5) and (6). $p_{se}(\cdot, \cdot) = p_e(\cdot, \cdot) + p_e(\cdot, \cdot)$. The linear blending is possible since, in the BE formulation (see formula (4)), the function $p$ can be written as a sum of two

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$^1$If not specified, $g = 1$ for all the experiments in the paper.
Fig. 2. An example of selective sharpening using the BE (see details in the text), image size is 348x235 pixels.

(or more) parts that linearly multiplies the function $c$ and are linearly integrated. Figure 3 shows an example of simultaneous edge preserving smoothing and enhancement, with parameters $\eta_s = 1$, $\eta_c = 0.25$, $\sigma_c = 1$ and $\sigma_s = \sigma_e = 0.04$. As it can be seen, this setting removes the noise while, at the same time, slightly enhances ($\eta_c = 0.25$) the more prominent edges.

Fig. 3. An example of simultaneous edge-preserving smoothing and selective sharpening using the BE (see details in the text), image size is 135x168 pixels.

3. RESULTS AND DISCUSSION

An extensive comparison to the (many) other methods in literature is not possible due to paper length limit. However, we want to compare the proposed BE algorithm with a classical BF (both the domain function $c$ and range function $s$ are Gaussian). The test aims at comparing the two algorithm at the top of their possibility on a noise-removal task on synthetic data.

To this aim, we created a gray-scale image $I(x)$ composed of 20 pixels side steps each of 0.2 height, covering the image range from 0 to 1. Then we added zero-mean Gaussian noise in increasing standard deviation from $\sigma_N = 0.01$ to 0.2 in steps of 0.01 obtaining the images $I_{\sigma_N}(x)$. For each noisy image we estimate the optimal parameters of the BE and the BF that minimizes the following functional:

$$E(\hat{I}_{\sigma_N}, I) = \frac{\sum_{x \in I} (\hat{I}_{\sigma_N}(x) - I(x))^2}{|I|}$$

where $I$ is the image support and $\hat{I}_{\sigma_N}$ is the result of the filter applied to the noisy image $I_{\sigma_N}$. This functional is actually the RMS of the error estimate.

In the case of the BF we minimize the functional on just two parameters: the standard deviations of the Gaussian functions in the image domain $\sigma_c$ and in the intensity domain $\sigma_s$. For the BE, we have to estimate $\sigma_c$ as for the BF, plus the parameters of the function $p_s(\cdot, \cdot)$, $\eta_s$ and $\sigma_e$. This minimization is repeated on several trials for the same noise standard deviation $\sigma_N$. Figure 4 shows the average functional $E(\hat{I}_{\sigma_N}, I)$ respect to the standard deviation of the Gaussian noise $\sigma_N$.

Fig. 4. The minimized $E(\hat{I}_{\sigma_N}, I)$ for BF and BE respect to the input noise standard deviation $\sigma_N$.

It is interesting to note that the RMS errors of the BE and BF are very similar. This shows that the the proposed approach is as good as the BF in an edge-preserving de-noising task. Finally, Figure 5 shows an example in which the BE exhibits a noise removal capability that is equal to the BF while, thanks to the enhancement part, some details have much more contrast and sharpness, e.g. eyes, legs and mouth. In this case the parameters of the BF are $\sigma_c = 2.5$ and $\sigma_s = 0.3$, while the parameters for the BE are $\sigma_c = 2.5$, $\sigma_s = \sigma_e = 0.3$, $\eta_s = 1$ and $\eta_c = 0.5$.

4. CONCLUSION AND FUTURE WORK

In this paper we presented a method that can simultaneously perform edge-preserving smoothing and selective enhancement/sharpening in an efficient way. We shown that the method performs as good as the BF in the task of edge-preserving de-noising. The properties of the BE are supported by both empirical and theoretical results. Future works encompass an extensive testing and comparison against the state of the art.

A. BE REDUCES TO A GAUSSIAN FILTER

In the case of $g = 1$, $c(\xi, x) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp\left(-\frac{|x-\xi|^2}{2\sigma_c^2}\right)$ and $p(\cdot, \cdot) = f(\xi) - f(x)$, using the linearity of the integral, and
grouping \( f(x) \), formula (4) can be written as:

\[
j(x) = f(x) \left( 1 - \int_{\Omega(x) \setminus x} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} d\xi \right) + \int_{\Omega(x) \setminus x} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} f(\xi) d\xi
\]

Since the Gaussian function, in this case, is also a distribution, the first addendum is the product of the value \( f(x) \) and the coefficient of the Gaussian distribution in the origin; for this reason, it can be incorporated in the second integral, giving the formula of a Gaussian filtering:

\[
j(x) = \int_{\Omega(x)} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} f(\xi) d\xi.
\]

**B. BE REDUCES TO A DOG SHARPENING**

In the case of \( g = 1 \), \( c(\xi, x) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp \left( -\frac{|x-\xi|^2}{2\sigma^2} \right) \) and \( p(\cdot, \cdot) = f(x)-f(\xi) \), using the linearity of the integral, formula (4) can be written as:

\[
j(x) = f(x) + f(x) \int_{\Omega(x) \setminus x} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} d\xi + \int_{\Omega(x) \setminus x} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} f(\xi) d\xi
\]

Again, since the Gaussian function, in this case, is also a distribution, the second addendum can be rewritten as \( f(x) \left( 1 - \frac{1}{\sigma_c \sqrt{2\pi}} \right) \) and finally, with some simple algebra, we obtain:

\[
j(x) = f(x) + \left( f(x) - \int_{\Omega(x)} \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{|x-\xi|^2}{2\sigma^2}} f(\xi) d\xi \right)
\]

Here, the quantity between brackets can be seen as a Difference of Gaussians were the first Gaussian is reduced to a delta distribution centered in \( x \).

**5. REFERENCES**


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**Fig. 5.** An example of simultaneous edge-preserving smoothing and sharpening, image size is 640x853 pixels.