Impulse noise removal with gradient adaptive neighborhoods

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Abstract. A new effective algorithm of impulse noise suppression is proposed. Conventional filtering schemes usually utilize a fixed shape for the moving window, such as a rectangle or circle. In contrast, the proposed algorithm exploits a signal-dependent shape for the moving window. We suggest a simple adaptive algorithm of impulse noise detection in monochrome images that takes into account the size of signal gradient neighborhoods and image statistics. Experimental results show superior performance of the proposed algorithm compared to that of conventional algorithms in terms of both subjective and objective criteria.

Subject terms: image processing; nonlinear adaptive filtering; impulse noise; spatially adaptive neighborhood.

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1 Introduction

The goal of many image processing tasks is to recover an ideal high-quality signal from data that are degraded by impulsive noise. Since the human visual system is very sensitive to high amplitude of noise signals, such degradation of an image can result in subjective loss of information. Impulse noise is usually characterized by abrupt alterations of intensity values in the input signal. Various algorithms have been proposed for impulse noise removal.1–7 Basically, most of these algorithms are based on the calculation of rank-order statistics.8 In the case of the outlier method,9 first an average of gray levels in a filter window around the center pixel is calculated. Next, the absolute difference between the average value and the gray level of the center pixel is calculated and compared with a threshold value. The analyzed pixel is marked as a noise peak if the difference is greater than a given threshold. The method of fuzzy techniques10 exploits a similar idea; however, the average is replaced by the median operator. Other algorithms utilize local derivative and Laplacian of a signal for detection of impulses. Note that most existing techniques do not consider groups of noise impulses (clusters). Actually, the \( N \times N \) median filter removes all impulse noise clusters, with sizes less than \( \lceil \frac{N \times N + 1}{2} \rceil \) pixels. For instance, the \( 3 \times 3 \) median filter removes a noise cluster if its size is less than five pixels. The probability of noise cluster occurrence can be estimated using equations given in other papers.11,12 The median filter is usually implemented uniformly across an image, and it tends to modify pixels that are undisturbed by noise. Consequently, suppression of all noise impulses is often at the expense of blurred and distorted features. Furthermore, the fine structures of an image can also be erased with the median filter. Effective techniques consist of two steps. First, a filter detects corrupted pixels, and then a noise cancellation scheme is applied only to detected noisy pixels.

Recently, nonlinear filters for monochrome images with a signal-dependent shape of the moving window have been proposed.13 This approach with signal-dependent moving windows was utilized to design a new filter for suppressing impulse noise in highly corrupted color images.11,12 We replaced conventional sliding windows, which usually have fixed shape and size, by a spatially adaptive neighborhood (SAN). The SAN is defined as a set of image pixels that are geometrically close to the tested pixel and concurrently close to the value of the central pixel.

In this work, a modified SAN called a gradient adaptive neighborhood (GAN) is suggested. The performance of the proposed algorithm in test scenes is compared with those of conventional schemes. The comparisons are made in terms of objective and subjective criteria.

The work is organized as follows. Section 2 presents a helper function analysis, which simplifies the entire description of the method. In Sec. 3, we introduce a concept of GAN. New detection and filtering algorithms based on the GAN concept are described in Sec. 4. Experimental results with test images are presented and discussed in Sec. 5. Section 6 summarizes our conclusions.

![Fig. 1](image-url) (a) Test image corrupted by “salt and pepper” noise (12%), and (b) its helper function based on the local \( 3 \times 3 \) median operator.
2 Analysis with Helper Function

Detection of noise impulses is the first step of efficient outlier suppression techniques. First, we introduce a helper function \( h_{i,j} \). Let \( v_{i,j}, \hat{v}_{i,j} \) be an original (not distorted) image, a noisy image, and a processed image, respectively. Here, \( \{i,j\} \) are the image matrix indices. The codomain \( \Lambda \) of an ideal helper function can be divided into two non-overlapping sets \( \Lambda^d \) and \( \Lambda^u \) as follows: if \( h_{i,j} \in \Lambda^d \), then the pixel is distorted with impulse noise; if \( h_{i,j} \in \Lambda^u \), then the pixel is uncorrupted. In real applications an approximated helper function is used. Let us consider an example of the helper function, which is based on a median operator,

\[
h_{i,j} = \begin{cases} \text{MED} (\hat{v}_{i+r,j+p}) - \hat{v}_{i,j}, & \text{if } D_{i,j} = \text{TRUE} \\ \hat{v}_{i,j}, & \text{if } D_{i,j} = \text{FALSE} \end{cases}
\]

(1)

where MED denotes the median operator and \( R \) is the size of a sliding window.

A realization of the median-based helper function is shown in Fig. 1(b). The partition of the helper function codomain into two parts gives the following detection rule:

\[
D(h_{i,j}) = \begin{cases} \text{TRUE}, & \text{if } h_{i,j} \in \Lambda^d \\ \text{FALSE}, & \text{if } h_{i,j} \in \Lambda^u \end{cases}
\]

(2)

Using this rule, the mask of detected impulses \( D_{i,j} = D(h_{i,j}) \) can be easily formed. Next, a noise cancellation scheme is applied only to detected noisy pixels. In other words, all values \( \hat{v}_{i,j} \) of the detected noisy pixels are replaced by their estimates \( \hat{v}_{i,j} \). The output of the proposed algorithm can be written as follows:

\[
\hat{v}_{i,j} = \begin{cases} \text{MED} (\hat{v}_{i+r,j+p}), & \text{if } D_{i,j} = \text{TRUE} \\ \hat{v}_{i,j}, & \text{if } D_{i,j} = \text{FALSE} \end{cases}
\]

(3)

where \( R=1 \) if the \( 3 \times 3 \) sliding window is used. The pixels associated with outliers after the detection \( D_{i,j}=\text{TRUE} \) are excluded from the median estimation. In rare cases when the detection values \( D_{i,j}=\text{TRUE} \) for all pixels of the sliding window, then the parameter \( R \) is increased.

The result of such processing can be evaluated with a mean squared error (MSE). The MSE is defined as

\[
\text{MSE} = \frac{1}{|\Omega|} \sum_{i,j \in \Omega} (v_{i,j} - \hat{v}_{i,j})^2,
\]

(4)

where \( \Omega \) is the domain of the indices, and \(|\Omega|\) is the number of pixels in the domain. Equation (4) can be rewritten as follows:

\[
\text{MSE} = \frac{1}{|\Omega|} \left[ \int_{\lambda \in \Lambda^d} \sum_{i,j \in \Omega} (v_{i,j} - \hat{v}_{i,j})^2 \delta(h_{i,j} - \lambda) d\lambda + \int_{\lambda \in \Lambda^u} \sum_{i,j \in \Omega} (v_{i,j} - \hat{v}_{i,j})^2 \delta(h_{i,j} - \lambda) d\lambda \right],
\]

(5)

where \( \delta(x) \) denotes the Dirac delta function.

**Fig. 2** MSE and PSNR versus detection threshold \( h_d \) for the median-based helper function of the corrupted test image (Fig. 1).

Since the processed image is a finite set of pixels, the codomain \( \Lambda \) is also a finite set of the helper function values, say \( \Lambda = [h_{\min}, h_{\max}] \). Let us sort all elements of the set \( \Lambda \) in ascending order with respect to their values; that is, \( h_{\min} \leq h_0 \leq h_1 \leq \ldots \leq h_n \leq h_{\max} \). The optimal way to divide the codomain into two subsets can be written as

\[
D(h_{i,j}) = \begin{cases} \text{TRUE}, & \text{if } h_{i,j} \equiv h^*_d \\ \text{FALSE}, & \text{otherwise} \end{cases}
\]

(6)

where \( h^*_d \) is a threshold parameter.

Now Eq. (4) can be rewritten as a function of \( h^*_d \); that is,

\[
\text{MSE}(h_d) = \frac{1}{|\Omega|} \left[ \sum_{\lambda = h_{\min}}^{h_d} \sum_{i,j \in \Omega} (v_{i,j} - \hat{v}_{i,j})^2 [h_{i,j} = \lambda] + \sum_{\lambda = h_d}^{h_{\max}} \sum_{i,j \in \Omega} (v_{i,j} - \hat{v}_{i,j})^2 [h_{i,j} = \lambda] \right],
\]

(7)

where \([\cdot]\) denotes the following function: 1 if the statement in parentheses is true; and 0 otherwise.

The MSE function (for a given test image and known impulsive noise positions) depends only on the threshold \( h_d \). An example of the MSE as a function of \( h_d \) is shown in Fig. 2. Here, \( \text{MSE}(h_{\min}) \) and \( \text{MSE}(h_{\max}) \) correspond to the result of filtering without impulse detection (in our case, the \( 3 \times 3 \) median filter is used) and to the corrupted image, respectively. In the plot we show an optimal value of the threshold \( h_d \) for the fuzzy filter and the ideal detection level of the MSE for the processed image when the noise map is known. The peak signal-to-noise ratio (PSNR) is often used as a quality measure in video processing,

\[
\text{PSNR} = 10 \log_{10} \frac{\text{max}^2(v_{i,j})}{\text{MSE}} = 10 \log_{10} \frac{255^2}{\text{MSE}}.
\]

(8)

The PSNR measured in decibels is also shown in Fig. 2. To utilize the described detection scheme, the following problems should be solved: how to find an optimal helper function and how to locate an optimal threshold \( h_d \) for this function with respect to the MSE. For instance, in Fig. 2 the graph is plotted using an uncorrupted image, which is available only in computer simulation. In real applications the
optimal helper function and threshold \( h_j \) can be computed using only a noisy image. Numerous computer experiments were carried out. We obtained that actually the optimal threshold \( h_j \) depends on the standard deviation of a signal and the probability of noise.

Basically, local filters with a fixed shape hardly recognize fine structures such as thin lines. To overcome this drawback, we suggested\(^{11-13}\) to replace conventional sliding windows that usually have fixed shape and size by spatially adaptive neighborhoods. Outlier detection that is based on a helper function with a fixed shape of the sliding window also possesses the same drawback as local filters. In this case, pixels belonging to a fine structure of an image usually have the same values of the helper function as those of impulse noise. In the next section, we propose a modification of the SAN, which helps us to obtain an excellent performance for noise detection. For instance, in Fig. 2 the minimal value of the MSE using the median-based helper function equals 85.7. If a helper function is based on the proposed modified SAN, then the minimum of the MSE is decreased to 61.3.

3 Gradient Adaptive Neighborhood

A concept of gradient adaptive neighborhoods is considered. We introduce the GAN as follows.

Gradient adaptive neighborhood is defined as a set of pixels of an image that satisfies the following conditions.

- Any two pixels of the neighborhood are gradient connected. This means that there exists a path (based on four-neighbor or eight-neighbor adjacency) between these two pixels in the spatial neighborhood.
- All pixels of the path satisfy the following condition: \( |v_k - v_{k\pm1}| \leq \partial \), where \( \partial \) is a given threshold, and \( \{v_k, v_{k\pm1}\} \) are values of any pair of adjacent pixels on the path.

Figure 3(a) illustrates two paths between pixels \( v_{0,2} \) and \( v_{0,5} \) of gradient connected neighborhoods. Figure 3(b) shows two formed gradient adaptive neighborhoods.

Let us describe pixels of an image as cells. The edges that separate these cells may be represented as partitions or thin dams [see Fig. 3(b)]. The heights of partitions are proportional to the absolute difference of adjacent pixels. When any pixel of the image is flooded by letting water rise to a fixed level (in our case, it is the threshold \( \partial \)), then several noncommunicating pools are formed (with unique levels of water for each pool).

In our previous work\(^{11-13}\) we formed the SAN as a spatially connected region with the central pixel of a moving window. Note that formation of the SAN is a time-consuming procedure, depending on the size of the moving window and the signal to be processed. Segmentation on the basis of the noncommunicating pools model is unique for any chosen image and a fixed threshold \( \partial \). The average number of operations needed to form the GAN of a real image is three to four simple arithmetic and logic operations per pixel.

4 Design of Outlier Detector

In this section, a simple detector of impulse noise in monochrome images is proposed. Assume that the size of the GAN of a noise cluster is smaller compared to that of details to be preserved after filtering. Therefore, impulsive noise can be detected by checking the size of the cluster. A similar idea is exploited for the GAN. First, the helper functions are calculated,

\[
\theta_{i,j}(\partial) = S_{i,j}(\partial),
\]

where \( S_{i,j} \) is the number of pixels included in the GAN, \( \partial \) is the parameter of the GAN, and the pixel \( \{i,j\} \) belongs to the same cluster.

For optimal detection, two parameters \( h_d \) and \( \partial \) need to be computed. Our computer experiments have shown that the optimal pair of the parameters depends implicitly on the statistical characteristics of a signal and noise. To simplify the detection algorithm, we propose to eliminate the direct dependence of the helper function on \( \partial \). First, we form the GANs for each pixel with different values of \( \partial = 1, 2, \ldots \) while the size \( S_{i,j}(\partial) \) reaches a predetermined value, say \( S_c \), and then the helper function is calculated as follows:

\[
\theta_{i,j}(S_c) = \arg \min_{\partial} S_{i,j}(\partial) = S_c.
\]

Note that the obtained helper function is almost insensitive to the parameter \( S_c \). Extensive computer simulations have shown that the noise detection with \( S_c = 15 \) for noise percentage from 0 to 20% and \( S_c = 18 \) for noise percentage from 20 to 35% yields minimum errors for various distributions of impulse noise. It is interesting to note that in real applications, the helper function is determined after 7 to 12 steps of \( \partial \) per pixel.

The second needed parameter \( h_d \) can be approximately calculated in the following way,

\[
\hat{h}_d = \text{STD}(\hat{v}_{i,j}) = \text{STD}(v_{i,j}),
\]

where \( \text{STD}(\hat{v}_{i,j}) \) is the trimmed standard deviation of a noisy image. If parameters of impulse noise are known, then this statistic can be evaluated as

\[
\text{STD}(\hat{v}_{i,j}) = \left[ \frac{1}{(1-p)|\Omega|} \sum_{q=0}^{255} (H_q - \bar{H}_q)^2 \right]^{1/2},
\]

where \( H_q \) and \( \bar{H}_q \) are the histogram of a noisy image and the histogram of noise impulses, respectively; \( \bar{H}_{i,j} \) is the input data mean value; and \( p \) is the probability of impulsive noise.
The parameter $h_d$ depends also on many other factors, including the image content. We propose the following algorithm to estimate this parameter. If parameters of impulse noise are known, then the trimmed standard deviation of the noisy image is calculated with Eq. (12). A rough estimate $h_d$ of the parameter is calculated with Eq. (11). Outliers are detected with Eq. (6), and the masked median filtering given in Eq. (3) is carried out. We suppose that the filtered image $\hat{v}_{ij}$ possesses approximately the same statistical parameters as the original image $v_{ij}$. Now computer simulation with the filtered image $\hat{v}_{ij}$ and with a realization of impulse noise is performed. Note that if the parameters of the noise are not available on this stage of the algorithm, they can be estimated from the difference between the noisy and filtered images. The discrete function MSE as a function of $h_d$ is formed using Eq. (7). Finally, the sought parameter can be found as

$$h_d = \arg \min_{k|\Delta \leq h_d \leq \Delta} \{\text{MSE}(k)\},$$

where $\Delta$ sets an interval around $\hat{h}_d$ (it is experimentally set to 5).

The complete algorithm of impulse noise detection and removal consists of the following steps.

- For each image pixel $(i,j)$, form GANs with different $\sigma$ and compute the helper function $h_{ij}$ using Eq. (10) with an appropriate value of $S_c$.
- Calculate a rough estimate $\tilde{h}_d$ using Eqs. (11) and (12).
- Detect outliers with $\tilde{h}_d$ using Eq. (6) and perform the masked median filtering with Eq. (3).
- Carry out computer simulation with the filtered image and with a realization of impulse noise, and find the optimal parameter $h_d$ with Eq. (13).
- Detect outliers with the optimal parameter $h_d$ with Eq. (6) and perform the final masked median filtering with Eq. (3).

5 Experimental Results

Computer experiments are carried out to illustrate and compare the performance of conventional and proposed algorithms. How well, relative to the other filters, does each perform in terms of noise removal and preservation of fine structures? It is difficult to define an error criterion to accurately quantify image distortion. In this work, we base our comparisons on the MSE, and subjective visual criterion.

In our computer experiments, corrupted images are generated using both “salt and pepper” impulse noise (two intervals $[0, 12]$ and $[243, 255]$ with equal probabilities are used) and random-valued impulse noise (random values uniformly distributed in $[0, 255]$ is used). Figure 4(a) shows the original test image. The test image degraded due to “salt and pepper” impulse noise is given in Fig. 4(b). The probability of independent noise impulse occurrence is 20%. Figures 4(c) and 4(d) show the filtered images obtained from the noisy image in Fig. 4(a) with the conventional $3 \times 3$ median filter (MF) and the proposed filter (GANF), respectively. Note that the proposed filter using spatial pixel connectivity has a strong ability in impulse noise suppression and a very good preservation of fine structures and details. The visual comparison of the filtered images shows that the filtered image with the MF is much smoother than the output image after filtering with the proposed method. Note that fuzzy methods also work better than the conventional median filter. However, their usefulness is limited to cases with a low probability of impulse noise (about 10%).

Figures 5(a) and 5(b) show the original image and the same image corrupted with impulse noise. In this case, the noise is uniformly distributed between 0 and 255. The probability of independent noise impulse occurrence is 10%. Figures 5(c) and 5(d) show processed images with the fuzzy filter (FF) and the proposed filter, respectively. The visual comparison of the filtered images shows that outlier detection with the fuzzy filter is obviously worse than that with the proposed method.

Next, we carry out a series of computer experiments with “salt and paper” impulse noise to make a numerical comparison of the proposed algorithm with the conventional median filter and with the fuzzy filter. The performance of the filters is presented in Table 1. Filtering quality is measured in decibels.

Thus, the analysis of computer experiments shows the proposed filter has a strong ability in impulsive noise reduction and a very good preservation of fine structures and details.
6 Conclusion

We present a new algorithm for detection and suppression of impulsive noise. The proposed technique takes into account three important factors for image filtering, i.e., noise attenuation, edge preservation, as well as detail retention. The conventional filtering schemes utilize a fixed shape of the moving window, while the proposed spatially connected filter works with a moving window of signal-dependent shape. Extensive simulation results in test images show superior performance of the proposed filtering algorithm compared to conventional schemes in terms of both subjective and objective evaluations. Finally, we note that optical implementations of adaptive algorithms cannot be fast. However, currently optical information often passes through digital channels, and special fast processors, which are included in signal transmission schemes, may perform adaptive algorithms in real time. Furthermore, hybrid opto-digital systems for impulse noise removal can also be designed.

| Table 1 Impulse noise suppression with various filters and different parameters of impulse noise. |
|---------------------------------|-----|-----|-----|-----|
| Type of filters                | P=0.01 | P=0.08 | P=0.15 | P=0.25 |
| MF                             | 289.8 (23.5 dB) | 315.5 (23.1 dB) | 356.4 (22.6 dB) | 486.7 (21.2 dB) |
| FF                             | 18.3 (35.6 dB) | 67.4 (29.8 dB) | 165.9 (25.9 dB) | 398.2 (22.1 dB) |
| GANF                           | 5.67 (40.6 dB) | 49.6 (31.2 dB) | 112.3 (27.6 dB) | 239.6 (24.3 dB) |
| Ideal detection                | 2.61 (43.9 dB) | 20.9 (34.9 dB) | 55.6 (30.6 dB) | 81.5 (28.9 dB) |

Fig. 5 (a) Test image, (b) test image degraded due to impulse noise (10%), (c) filtered image with fuzzy filter, and (d) filtered image with proposed filter.

References


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