

Eigensnakes for vessel segmentation in angiography

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Abstract

In this paper we introduce a new deformable model, called eigensnake, for segmentation of elongated structures in a probabilistic framework. Instead of snake attraction by specific image features extracted independently of the snake, our eigensnake learns an optimal object description and searches for such image feature in the target image. This is achieved applying principal component analysis on image responses of a bank of gaussian derivative filters. Therefore, attraction by eigensnakes is defined in terms of classification of image features. The potential energy for the snake is defined in terms of likelihood in the feature space and incorporated into a new energy minimising scheme. Hence, the snake deforms to minimise the mahalanobis distance in the feature space. A real application of segmenting and tracking coronary vessels in angiography is considered and the results are very encouraging.

Keywords. Snakes, Principal Component Analysis, Statistical Learning, Segmentation, Angiography.

1. Introduction

The main areas of research in computer applications for angiography during the past 15 years have been devoted to geometric and densitometric methods to automate quantitative analysis of coronary arteriograms. The first steps include assessment of coronary lesion severity [10] in individual segments followed by a growing interest in automated identification and analysis of entire coronary tree. Over the last few years, attention has been directed to research towards 3D reconstruction from biplane projection [2, 9], to improve measurements of small vessels [11] to mix data coming from different imaging devices [7] and to obtain 3D dynamic models [13]. On the other hand, computer and communication technologies are growing at incredible speed, increasing performance with parallel decreasing in prices. Such phenomena open new research fields with clear

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application feasibility that few years ago should have been only of theoretical scope. Correct vessel segmentation is a key issue in any automatic analysis task. To extract and use the information present in the coronary image, many conventional methods have been proposed, although ambiguities and artifacts make the segmentation process highly dependent on heuristics (parameter tuning). Despite the increasing quality of the imaging equipment, the computer analysis task still remains non-trivial. Hence, the impact of any improvement of the segmentation step is important. Two main strategies to segment the coronary tree have been reported:

- *Scanning* consists of edge or ridge extraction usually by a mask convolution. The second step implies recognition of the vascular structure by chaining the centerline points while excluding heuristically noise points. Most of the reported image feature detectors are conventional ones: Laplacian of a Gaussian [6], hat transform [15], ridge detector based on level-set theory [4], etc.
- *Tracking* begins at an a-priori known position of the vessel in the image. In a single pass operation, feature extraction and vessel structure recognition are performed. By its own nature, a tracking strategy is computationally more efficient than scanning. In [12], given a starting point and direction, a line profile extracted some pixels ahead in that direction is used to compute the centerline of a vessel and a new forward direction for the tracking. In [1], a curve sampling profile is extracted and used for tracking the whole tree.

Conventional image feature detectors used in scanning are too general for the purpose of vessel detection in angiography bringing too many false responses. On the other hand, tracking strategy relies on simple densitometric features that are not enough to discriminate the vessel appearance. Moreover, the tracking strategy needs a continuous set of image features; too strong constraint for angiographies.

To cope with these shortcomings we define a new statistic model called eigensnake that integrates snake technique as a global segmentation and interpolation method combined with statistic image feature learning. Segmentation using snakes is a well-known technique that comprises two steps; feature detection using a scanning method to construct a potential map followed by an energy minimisation of the snake curve towards minima of the potential (the image features of interest). The object recognition step is built in the curve shape deformation. Taking advantage of this property, our proposal is to reduce the method to only one step. The idea is to specify the feature detector depending of the target object avoiding the map construction of the conventional snakes. The eigensnake has defined its external energy as a function of the Mahalanobis distance of the image features located in the snaxel position to a learned statistical vessel description. The vessel description is obtained from object responses of gaussian derivative filters over different scales. To obtain filter response invariant to the vessel orientation, we project the filter output along the direction of grey-level variance [14]. Given that each point of the vessel is represented by a set of filter responses, a dimensional reduction is carried out by means of Principal Component Analysis to define a reduced feature space. A likelihood map is constructed to illustrate the separability of vessel and no vessel representations. The snake deforms using as external energy the Mahalanobis distance between the image features in the pixels under analysis and the learned feature projecting them into the reduced space. This process guides the snake towards the vessel centrelines. The paper is organised as follows: section two is a summary of snake principles, section three explain the learning process, section four focuses on the external energy building process, section five presents the results and finally, conclusions are given.

2. Snakes

Snakes are physics-based models, defined in Newton mechanics that deform under external and internal forces. These models are represented as elastic curves with associated energy. External energy is defined as a function of the curve distance to the image features of interest. Internal energy depends on the smoothness and continuity of the model shape. The segmentation by snakes is defined as an energy- minimisation problem. The snake deforms as close as possible to the image features of interest minimising its external energy, while keeping its shape as smooth as possible minimising its internal energy. Representing parametrically the position of the snake as $\mathbf{v}(s) = (x(s), y(s))$, the energy functional of the snake is written as follows:

$$E_{snake} = \int_0^1 E_{int}(\mathbf{v}(s)) + E_{ext}(\mathbf{v}(s)) ds \quad (1)$$

where $E_{int}(\mathbf{v}(s))$ is the internal energy and $E_{ext}(\mathbf{v}(s))$ is the external energy. Snake energy 1 is minimised by Euler-Lagrange equation yielding:

$$-\frac{\delta}{\delta s}(\alpha \mathbf{v}_s) + \frac{\delta^2}{\delta s^2}(\beta \mathbf{v}_{ss}) + \nabla E_{ext}(\mathbf{v}(s)) = 0 \quad (2)$$

The external force ∇E_{ext} makes the snake to approach and lock on image features (minimising the external energy). To define the external energy a detector of image features is applied and a potential map as a function of the distance to the extracted image features is built [8, 3]. Our aim is to define a new statistical external energy so that the snake is attracted only by image features matching statistically constructed vessel description.

3 Learning the feature

To define the statistical external energy, a learning process is carried out invariant to vessel orientation applying the structure tensor. A set of generalized filters projected on the first eigenvector of the structure tensor is used to construct the probabilistic model of the vessel.

3.1 Extraction of structure orientation

A structure tensor [14] is used to learn vessel appearance in the direction of maximal grey-level variation (perpendicular to the vessel). A tensor J_ρ is obtained by a tensor product of the image gradient ∇u_σ smoothed by a gaussian kernel: $k_\rho(x, y) = \frac{1}{2\pi\rho^2} \exp(-\frac{|x|^2+|y|^2}{2\rho^2})$. The expressions for the smoothed image and the structure tensor are as follows: $u_\sigma(x, t) = (K_\sigma * u(\cdot, t))(x)$
 $j_\tau(\nabla u_\sigma) = K_\tau * (\nabla u_\sigma \nabla u_\sigma^T) \quad (\tau \geq 0)$

The eigenvalues of the tensor describe the contrast variation in the eigendirections \mathbf{e}_1 and \mathbf{e}_2 . The eigenvector \mathbf{e}_2 associated to the lower eigenvalue gives the orientation of the lowest fluctuation, detecting the vessel flow, while the first eigenvector describes the normal direction used in the learning process (fig. 1(a)).

3.2 Derivative projections

A bank of Gaussian derivative filters $\frac{\delta^k K_\rho}{\delta x^{k_1} \delta y^{k_2}}$ at different scales ρ is used to obtain a statistic vessel description. Note that different scales are necessary to cope with the vessel diameter variability while using different derivatives allows us to generalise edge- crest- and valley detectors. We use the coherence direction of the vessel structure to orient the filters. We define a mapping of the image pixels to the space of filter responses as follows:

$$F : I \rightarrow \mathbf{R}^n$$

$$(x, y) \rightarrow \mathbf{f} = (f_1, \dots, f_n)$$

Each sample f_i is a filter output $u_{k\rho}$ in a vessel pixel oriented by the eigenvector \mathbf{e}_1 :

$$u_{k\rho}(x, y) = \frac{\delta^k K_\rho}{\delta x^{k_1} \delta y^{k_2}} * u(x, y) \quad k = k_1 + k_2$$

$f_l = u_{k\rho}(x, y) \cdot \mathbf{e}_1, l = 1 \dots n$. A matrix is built where each row is a sample along the vessel. Given a set of training points (fig. 1(b)) we get their filter responses $f_l, l = 1, \dots, n$ and construct the training data matrix $\mathbf{D}m$. In fig. 2(a) first derivative projection with $\rho = 9$ is showed. Figure 2(b) depicts a training data matrix.

3.3 Dimensional reduction

Using Principal Component Analysis [5] a dimensional reduction is carried out as follows: $W : R^n \rightarrow R^l \quad (l < n) : \mathbf{f} \rightarrow y$. To obtain the principal components, we compute the eigenvectors of the covariance matrix Σ of $\mathbf{D}m_{m \times n}$. The eigenvectors are sorted according to their associated eigenvalues (variances) and form the columns of matrix W . Such reduced space is used to measure the distance of an image feature to the learned ones. The measure can be regarded as a likelihood function giving the probability of each pixel belonging to a vessel category. Figure 3(a) shows the training data projected onto the first two eigenvectors. Fig. 3(b) shows all image features projected onto the first two eigenvectors, the center of the cluster contains the learned data of fig. 3(a). Figure 3(c) shows the learned data projected onto the principal component coordinate system.

4 A probabilistic energy-minimizing scheme

Our aim is to make the snake to be attracted by image features corresponding to the statistical description of the object. To this purpose, we define the external energy of the snake as a function of the Mahalanobis distance of the projected image features x to the centre μ of the learned cluster. The Mahalanobis distance [5] is computed in the reduced feature space, defined as follows:

$$E_{ext}(\mathbf{v}) = D_I^2(\mathbf{v}, \mu) = (WFI(\mathbf{v}) - \mu)^T \Lambda^{-1} (WFI(\mathbf{v}) - \mu) \quad (3)$$

where Λ is a diagonal matrix containing the eigenvalues of the training covariance matrix and $I(\mathbf{v})$ is a vector representation of the image neighbourhood around the snake pixels. Using 2 and the gradient of the probabilistic external energy in 3, we get a new energy minimising scheme for the snake:

$$-\frac{\delta}{\delta s}(\alpha \mathbf{v}_s) + \frac{\delta^2}{\delta s^2}(\beta \mathbf{v}_{ss}) + (\cos \varphi, \sin \varphi) \frac{\delta D}{\delta \mathbf{e}_1} = 0 \quad (4)$$

where $\frac{\delta D}{\delta \mathbf{e}_1} = (WFI(\mathbf{v}) - \mu)^T \Lambda^{-1} (W \frac{\delta F}{\delta \mathbf{e}_1} I(\mathbf{v}))$ and $\mathbf{e}_1 = (\cos \varphi, \sin \varphi)$ is the first eigenvector.

5 Results

To illustrate the viability of our eigensnakes, we consider a real application of detecting coronary vessels in angiographies. We tested our approach on 23 images and 5 different vessels. Using five scales for the filters with parameter $\rho = 9 \dots 13$ and derivatives up to third degree, we learned 130 points, and constructed a data matrix $D_{m \times n}, m = 130, n = 26$. Ought to the high number of pixels (samples) in any image, a dimensional space reduction by means of PCA is carried out from $n = 26$ to $l = 4$. In our case, the first four principal axes (eigenvectors) explain up to 99% of the variances, (fig. 3)(c).

For illustrative purposes a mahalanobis distance map is built projecting all image features onto the reduced space and measuring the distance to the training vessel cluster. The Mahalanobis distance map shows the snake convergence to a vessel. Due to the statistic learning, Mahalanobis distance is small mainly in vessel positions. As a result, false responses of vessel appearance are diminished and the snake does not suffer from local energy minima shortcoming. On the other hand, approaching the vessel, the Mahalanobis distance exponentially decreases driving the snake to lock on the vessel features. Figure 4(a) shows the probabilistic external energy map as a function of the Mahalanobis distance. In fig.4(b) a snake is used to segment a vessel. The snake has converged to the vessel in 30 iterations using the energy showed in figure 4(a). One can notice that in a real application the built-in map approach (4) is preferable to obtain faster energy-minimisation scheme avoiding explicit construction of likelihood map for the whole image.

6 Conclusions

In this paper we have proposed a new formulation of the energy-minimising scheme that allows statistically learning and detecting image features characterising different appearances of non-rigid elongated objects. Incorporating the statistical framework, the approach can be extended to the labelling task and to obtain the whole coronary tree using a likelihood matching.

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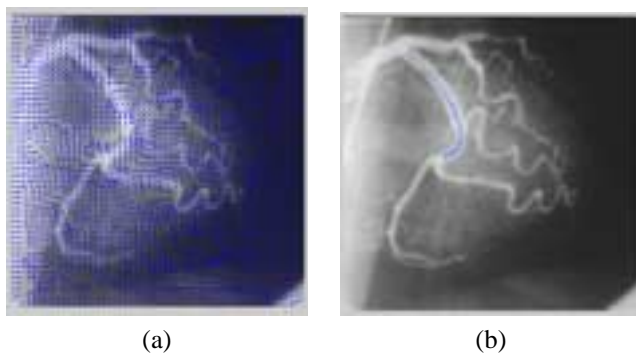


Figure 1. (a) First eigenvector, (b) Training image samples

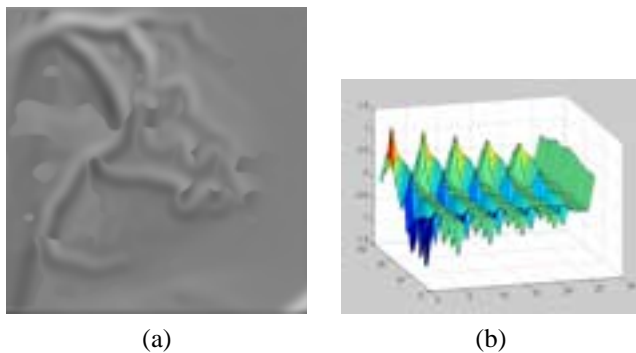


Figure 2. (a) First derivative, (b) Learned data matrix

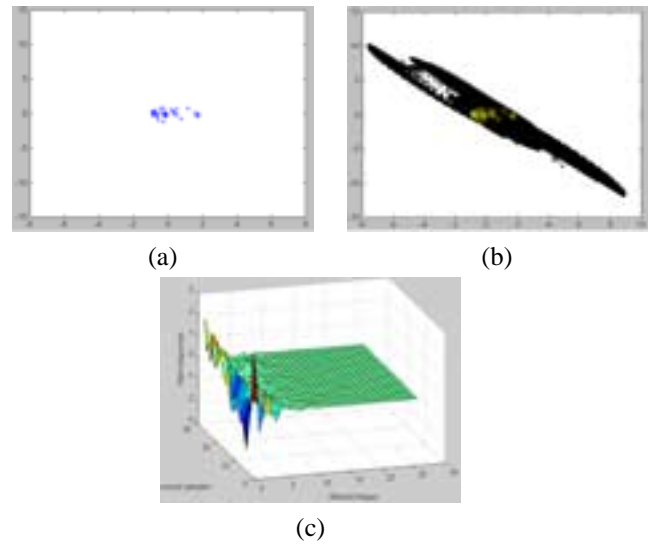


Figure 3. (a) Learned cluster, (b) Image projection, (c) Representation of training image feature in the feature space determined by principal axes

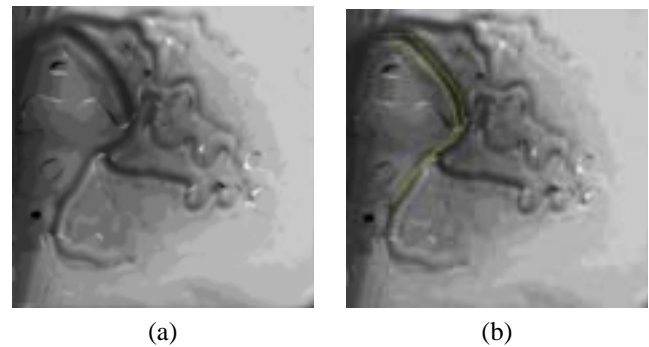


Figure 4. (a) Probabilistic external energy, (b) Snake segmentation

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